Vectors

Line $\underline{r} = \underline{a} + \lambda(\underline{b} - \underline{a})$

Coplanar condition $\beta\underline{a} + \nu\underline{b} + \mu\underline{c} = 0$

Scalar product $\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$\underline{a}.\underline{b} = a_x b_x + a_y b_y + a_z b_z$

Plane $(\underline{r} - \underline{a}).\underline{n} = 0$

Sphere, radius a, centre p $|\underline{r} - \underline{p}| = a$

Cylinder, radius a, unit normal p $|\underline{r} - (\underline{r}.\underline{n})\underline{n}| = a$

Cone, half angle α, tip at p, unit normal n $|\underline{r} - \underline{p}|\cos\alpha = (\underline{r} - \underline{p}).\underline{n}$

Vector area, area a, unit normal n $\underline{S} = a\underline{n}$

Cross product $|\underline{a} \wedge \underline{b}| = |\underline{a}||\underline{b}|\sin\theta$

Scalar triple product $\underline{a}.\underline{b} \wedge \underline{c} = -\underline{a}.\underline{c} \wedge \underline{b}$

Vector triple product $\underline{a} \wedge (\underline{b} \wedge \underline{c}) = (\underline{a}.\underline{c})\underline{b} - (\underline{a}.\underline{b})\underline{c}$

Complex Numbers

Roots $z^{\frac{1}{n}} = |z|^{\frac{1}{n}} e^{i\frac{\theta + 2m\pi}{n}}$

De Moivre $(\cos n\theta + \sin n\theta) = (\cos\theta + \sin\theta)^n$

Trigonometry $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

Hyperbolics $\cosh\theta = \frac{1}{2}(e^{\theta} + e^{-\theta})$

$\sinh\theta = \frac{1}{2}(e^{\theta} - e^{-\theta})$

$\cos\theta = \cosh i\theta \quad i\sin\theta = \sinh i\theta$

$\cosh\theta = \cos i\theta \quad i\sinh\theta = \sin i\theta$

To change from normal to hyperbolic identity, change functions and reverse sign of $\sinh^2$

Trigonometry

Addition $\sin(a + b) = \sin a\cos b + \sin b\cos a$

$\cos(a + b) = \cos a\cos b - \sin a\sin b$

Corollary $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

Integration

Trigonometric $\int \sec^2\theta d\theta = \tan\theta$

Hyperbolics $\int \operatorname{sech}^2\theta d\theta = \tanh\theta$

$\int -\operatorname{cosech}^2\theta d\theta = \coth\theta$

$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a}$

$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\frac{x}{a}$

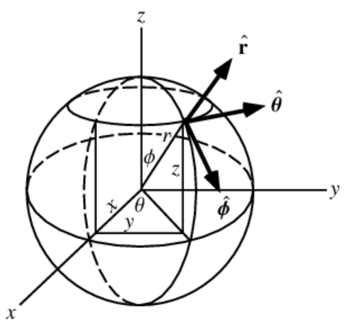
$\int \frac{a}{a^2 - x^2} dx = \tanh^{-1}\frac{x}{a}$

$\int \frac{-a}{x^2 - a^2} dx = \coth^{-1}\frac{x}{a}$

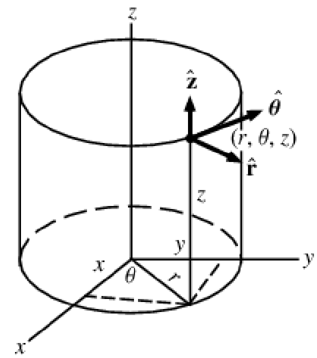
Tan integration $\int \frac{dx}{1 + x^2} = \tan^{-1} x$

Coordinate Systems

Spherical polar:



Cylindrical polar:



Spherical element $dV = r^2 dr \sin\theta d\theta d\phi$

Cylindrical element $dV = r dr d\theta dz$

Approximation

Taylor's theorem $f(x_1) \approx f(a) + ... + \frac{f^n(a)}{n!}(x_1 - a)^n$

Binomial expansion $(1 + x)^n = \sum_{r=0}^{r=n}\left(\frac{n!}{(n - r)!r!}x^r\right)$

Newton Raphson $x_{n+1} \approx x_n - \frac{f(x_n)}{f'(x_n)}$

$\varepsilon_{n+1} \approx \frac{f''(r)}{2f'(r)}\varepsilon_n^2$

Differential Equations

Linear form $\frac{dy}{dx} + P(x)y = Q(x)$

$\mu(x) = \exp(\int P(x)dx)$

$y = \frac{\int \mu(x)Q(x)dx}{\mu(x)}$

Real roots: $y = A\exp(\lambda_1 x) + B\exp(\lambda_2 x)$

Imaginary: $y = \exp(\alpha x)(C\cos\beta x + D\sin\beta x)$

Degenerate: $y = (A + Bx)\exp(\lambda x)$

Functions

Continuous at a — If defined at a and $\lim_{x \to a} f(x) = f(a)$ from any direction

Differentiable at a — If continuous at a and derivative continuous at a